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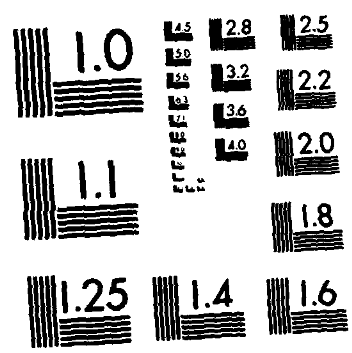

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An Upper Limit on Shepherding Satellites
at Saturn's Ring G

by

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U. of Iowa 86-30

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ABSTRACT

A simplified analytical theory of the absorption of energetic magnetospheric particles by an inert satellite is developed for the case in which the radius b of the satellite is much less than the equatorial gyroradius r_g (— sub g) of the particle which, in turn, is very much less than the radius r of the satellite's orbit. This theory and the observed particle-absorption signatures of Saturn's Ring G are used to establish an upper limit on shepherding satellites associated with the ring. The resulting upper limit, ignoring the absorption of the optically observed particulate matter, is b = 3.3 km (accurate to a factor of two) for a single satellite or $\sum_{i=1}^{n} b_i = 11.2\ km^2$ (accurate to a factor of four) for an assemblage of n satellites of various radii b_i. No shepherding satellites at Ring G have been detected optically.

INTRODUCTION

During Pioneer 11's passage through the inner radiation belts of Saturn on 1 September 1979, a number of distinctive absorption features were observed in curves of the radial dependence of the intensity of energetic, trapped particles [Van Allen et al., 1980a, 1980b; Simpson et al., 1980a, 1980b; Fillius et al., 1980; Fillius and McIlwain. 1980; Trainor et al., 1980; and McDonald et al., 1980]. One such feature was a well defined inflection (not a dip) in curves of the intensity of protons (E_p > 80 MeV) vs. radial distance. In the vicinity of this feature the inbound and outbound curves (at a time lapse of nearly four hours and at quite different longitudes) were virtually identical, suggesting that the feature was a time-stationary one. Van Allen et al. [1980b] designated this absorption feature 1979 S3 and labeled it as a "suspected [satellite] but interpretation of [absorption] signature ambiguous". The relevant ambiguity stems from the failure of such a limited set of data to distinguish the absorption macro-signature of one or more small satellites from that of a continuous circular ring of particulate matter [Van Allen,

1982]. The ambiguity was resolved to some extent in November 1980 by Voyager 1 images [Smith et al., 1981], which revealed a thin continuous ring of radius about 170,000 km, normal optical opacity $\approx 3 \times 10^{-5}$, and radial width about 500 km (after estimated correction for imaging "smear"). The Voyager investigators designated it Ring G. A detailed interpretation of the energetic particle data associated with Ring G has been reported [Van Allen, 1983]. The mean radius of Ring G was found to be 170,200 $\pm$ 900 km, in agreement with the optical value. Other conclusions of this work, which ignored the possibility of ring-associated satellites, were that "the particulates in Ring G have an effective radius $R \geqslant 0.035$ cm, an areal mass density $\sigma > 1.4 \times 10^{-6}$ g cm^{-2}, and an areal number density $n < 8 \times 10^{-3}$ cm^{-2}."

In related work, Goldreich and Tremaine [1979, 1982] suggested that small, as yet discovered, satellites might be closely associated with each of the narrow rings of Uranus and, if so, would act as agents (shepherds) for dynamically maintaining the observed distribution of particulate matter. This theoretical suggestion received resounding confirmation with the Voyager 1 discovery [Smith et al., 1981] of two small satellites (1980 S26 and 1980 S27) that apparently act as shepherds for Saturn's Ring F [Gehrels et al., 1980]. More recently, Smith et al. [1986] have observed two small satellites associated

with Uranus' Ring ϵ. However, there are many, many narrow rings at Jupiter, Saturn, and Uranus that have no thus-far detected shepherds associated with them. Saturn's Ring G is one such example; but, in this case, no comprehensive search in longitude for small (radii $\leq$ 10 km), nearby satellites was made by either Voyager 1 or Voyager 2 [B. A. Smith, private communication]. Hence, there is no definitive optical evidence either for or against the presence of such satellites at Ring G.

On the basis of a crude comparative analysis of the energetic particle intensity at Ring G and in the inner slot region (caused by Janus and Epimetheus), Van Allen [1983] reported that "there are no satellites having radii of the order of a kilometer or larger in or near Ring G." This comparison ignored all of the optically observed particulate matter in Ring G and attributed its entire absorption signature to one or more small satellites, thus attempting to establish extreme upper limits.

The present paper has the same purpose but employs an absolute, rather than a comparative, analysis and probably provides superior results.

ABSORPTION OF A MAGNETICALLY-TRAPPED, GYRATING PARTICLE BY A PLANETARY SATELLITE

The absorption of magnetospheric particles by planetary satellites has been discussed by a number of authors for various ranges of the relevant parameters [Singer, 1962; Mead and Hess, 1973; Mogro-Campero and Fillius, 1976; Thomsen, 1977; Van Allen et al., 1980a, 1980b; Thomsen et al., 1977; Van Allen, 1982, 1983; Rairden, 1980; Hood, 1981; Schardt and McDonald, 1983; Paonessa and Cheng, 1985; and Bell and Armstrong, 1986]. In its full generality, the absorption problem is a very complex geometrical one. Most of the above cited authors have treated it by approximate analytical methods. The two important exceptions are Rairden [1980] and Bell and Armstrong [1986], who used computerized sampling (Monte Carlo) methods.

The present treatment is specialized to the magnetosphere of Saturn and to the case in which the radius b of the (spherical, inert) satellite is very much less than the gyroradius r_g of the particle and in which r_g is, in turn, very much less than r, the radius of the satellite's orbit. It is further supposed that the magnetic moment of the planet is centered and aligned with its

rotational axis [Smith et al., 1980; Acuña and Ness, 1980; Acuña et al., 1980; and Connerney et al., 1984]; that the satellite is moving in a prograde circular Keplerian orbit of radius r in the planet's equatorial plane; and that the guiding center of the particle's helical motion drifts in longitude in a circle of approximately the same radius and oscillates in latitude between mirror latitudes $\pm \lambda_m$. With all of these simplifications, the absorption problem can be treated by a simple, analytical method having reasonable accuracy. Remarks on the effects of departures from the simplifying conditions will be made later. Other basic parameters are the kinetic energy E and species of the particle, its velocity vector $\vec{V}$, its equatorial pitch angle $\alpha_o = \arccos(\vec{V}_o \cdot \vec{B}_o / V_o B_o)$ (where $\vec{B}_o$ is the local magnetic field vector and the zero subscripts denote equatorial values), the equatorial values of its gyroradius r_g and gyroperiod T_g, and its latitudinal bounce period T_B. The relationship connecting α_o and λ_m is $\sin^2 \alpha_o = \cos^6 \lambda_m (4-3\cos^2 \lambda_m)^{-\frac{1}{2}}$. Extensive use is made of the formulae of Thomsen and Van Allen [1980]. For numerical examples of the Ring G problem, we assume that a representative particle is a proton E_p = 200 MeV and that a representative value of α_o is 70°. The other relevant parameters then have values as follows: r = 170,200 km; $T_g = 9.084 \times 10^{-2}$ s; $T_B = 3.085$ s; r_g = 2,305 km;

$\lambda_m = 9.6° = 0.168$ rad; and $V = 169{,}740$ km s^{-1}. The angular velocity ω of the satellite relative to the longitude of the particle's guiding center is 2.120×10^{-2} rad s^{-1} and thus $\omega r = 3{,}608$ km s^{-1}; and the interval of time T_E between encounters of the satellite with the gyrating particle is $2\pi/\omega = 296$ s. The overall results are relatively insensitive to the value of E_p within the energy range 80-300 MeV. In the course of the analysis, numerical values of calculated quantities are given in square brackets as an aid to quantitative understanding.

During a particular encounter, there are two opportunities for the particle to hit the satellite. The probability of a hit during each opportunity is now calculated.

The origin of the adopted Cartesian coordinate system is the point at which the guiding center of a particular gyrating particle pierces the equatorial plane; the origin moves in longitude at the same average rate as does the guiding center. The X-axis is tangent to the satellite's orbit, the Y-axis is directed to the center of the planet, and the Z-axis is parallel to the rotational axis of the planet. In this frame of reference the particle moves along a helical path on the surface of a truncated, circular tube of magnetic flux, whose axis at the equator is along the Z-axis, whose length is $\approx 2r\lambda_m$ and whose equatorial radius is r_g. The satellite drifts parallel to the X-axis through this surface with velocity $v_x = \omega r$, $v_y = o$, and $v_z = o$ and with its center at $y = y_o$ and $z = o$. A hit can occur only if $|y_o| \leq (r_g + b)$.

The X-Y plane section through the surface is shown in Figures 1(a) and 1(b). For the case shown in Figure 1(a), two separate opportunities for a hit occur within the two intervals of time during which the center of the satellite drifts from A to B (first to second contact) and from C to D (third to fourth contact). Let $\ell = \overline{AB} = \overline{CD}$. Then from the geometry of the figure,

$$\ell = \left\{ \sqrt{(r_g + b)^2 - y_o^2} - \sqrt{(r_g - b)^2 - y_o^2} \right\}, \quad \text{for } o \le |y_o| \le (r_g - b) \tag{1}$$

For the case shown in Figure 1(b), the two separate segments $\overline{AB}$ and $\overline{CD}$ merge into a single segment $\overline{AD}$; denoting $\ell = \overline{AD}/2$ for this case,

$$\ell = \left\{ \sqrt{(r_g + b)^2 - y_o^2} \right\}, \quad \text{for } (r_g - b) \le |y_o| \le (r_g + b) \tag{2}$$

A sample, composite plot of ℓ/r_g vs y_o/r_g is shown in Figure 2.

It is noted that:

$$\ell_{max} = 2\sqrt{b\, r_g} \quad \text{at} \quad |y_o| = (r_g - b) \tag{3}$$

and that the mean value of ℓ,

$$\langle \ell \rangle = \frac{\int \ell(y_o)\, dy_o}{\int dy_o} = \frac{\pi b}{(1 + b/r_g)} \approx \pi b \qquad (4)$$

with integration over the range $0 \leq |y_o| \leq (r_g + b)$ [cf. Schardt and McDonald, 1983]. The center of the satellite spends time Δt in tranversing ℓ where

$$\Delta t = \frac{\ell}{\omega r} \qquad (5)$$

and the radius of the circular patch of intersection between the cylindrical surface and the satellite during Δt varies from zero to b and back to zero with a mean value:

$$\langle b \rangle = \pi b/4, \qquad (6)$$

except for y_o nearly equal to r_g.

In the vicinity of the equator, the particle's motion may be visualized as composed of two components: a Z-component

$$V_z \approx V \cos \alpha_o \qquad [58{,}000 \text{ km s}^{-1} \text{ for } \alpha_o = 70°], \qquad (7)$$

and a circumferential component:

$$V_\varphi = \frac{2\pi\, r_g}{T_g} . \qquad [159{,}400 \text{ km s}^{-1}] \tag{8}$$

The random probability that the particle lies within the azimuth required for a hit during Δt is

$$p_1 = \frac{\Delta t}{T_g} = \frac{\ell}{\omega r\ T_g} . \tag{9}$$

If p_1 as calculated by (9) is less than unity for all values of y_o, then

$$\langle p_1 \rangle = \frac{\langle \ell \rangle}{\omega r\ T_g} = \frac{\pi b}{\omega r T_g} \qquad [9.6 \times 10^{-3}\ b, \text{ with } b \text{ in km}] \tag{10}$$

By (9) for $\ell_{max} = 2\sqrt{b\ r_g}$, p_1 exceeds unity if

$$b > 12 \text{ km}.$$

However, even if b exceeds this value by as much as a factor of 2, the mean value of $\langle p_1 \rangle$ as given by (10) is only 5 percent greater than its properly calculated value.

The independent (assuming no correlation between gyrophase and bounce phase) probability that the particle lies within $\pm\ \pi b/4$ of the equator at an arbitrary moment is

$$< p_2 > \; = \; \left(\frac{\pi b/2}{V \cos \alpha_o}\right) \left(\frac{2}{T_B}\right)$$

$$< p_2 > \; = \; \frac{\pi b}{V\, T_B \cos \alpha_o} \quad . \tag{11}$$

[For $\alpha_o = 70°$ and previously quoted values of V and T_B, $< p_2 > = 1.75 \times 10^{-5}$ b, with b in km.]

A typical mean value of Δt (for b = 5 km) is 0.0044 s; during this time interval the equatorial projection of the particle's trace in space (when near the equator) travels through an azimuthal arc of 0.30 radian or 0.048 of a circle. A typical time lapse between centers of the two opportunities is 1.1 s or about 12 gyro-periods of the particle, and therefore the probability for a hit in either opportunity, insofar as it is small, is independent of that in the other (except for singular cases). When $y_o \approx r_g$ the probabilities of a hit during the two opportunities are not statistically independent but for $b \ll r_g$ this qualification has a trivial effect on the overall analysis and I take the probability for a hit on a given encounter to be twice that for one of the two opportunities therein.

Thus the mean probability of a hit (averaged over y_o) during either of the two opportunities is:

$$<P> = <p_1> \cdot <p_2>$$

$$<P> = \frac{\pi^2 b^2}{\omega r V T_g T_B \cos \alpha_o} \tag{12}$$

For a complete encounter (two opportunities, assumed independent)

$$< P > = \frac{2\pi^2 b^2}{\omega r V T_g T_B \cos \alpha_o} \tag{13}$$

Putting my representative numerical values in (13)

$$< P > = 3.36 \times 10^{-7} b^2 \tag{14}$$

with b in km. Equations (13) and (14) are appropriate for $b \leq 20$ km and are the principal result of the foregoing analysis

$<P>$ in equations (13) and (14) may also be interpreted as the probability of a hit within T_E, the time interval between encounters. If there are n satellites of radii $b_1, b_2, \ldots b_n$ moving in random phases in the same orbit, then the probabilities of a hit are additive and

$$<P> = 3.36 \times 10^{-7} \sum_{i=1}^{n} b_i^2 . \tag{15}$$

Equation (15) shows that $\langle P \rangle$ per T_E is proportional to the sum of the cross-sectional areas of the n satellites, provided that all are large enough to absorb the specified particle in a single hit and small enough ($b \leq 20$ km) to meet the criterion for applicability of (13).

The number of encounters per day is $86{,}400/T_E = 292$ in the case at hand. Hence the mean lifetime τ of a representative particle against absorption by a satellite of radius b is given by

$$\tau = \frac{1.02 \times 10^4}{b^2} \text{ days} = \frac{28}{b^2} \text{ years} \tag{16}$$

or for n satellites having radii b_1, b_2, ... b_n,

$$\tau = \frac{28}{\sum_{i=1}^{n} b_i^2} \text{ years.} \tag{17}$$

In (14), (15), (16), and (17), b and b_i are in km.

COMMENTS ON VALIDITY AND ACCURACY OF EQUATION (13)

The derivation of equation (13), though novel in detail, is akin to those of previous authors [in particular, the derivation of Schardt and McDonald, 1983] but has, I think, greater clarity. Nonetheless, it is not exact even for $b \leqslant 10$ km. A discussion of its shortcomings follows.

By equation (11), the value of $\langle p_2 \rangle$ becomes equal to 1.0 if

$$\frac{\pi b}{V T_B \cos\alpha_o} = 1. \tag{18}$$

For $b = 10$ km, $V = 169{,}740$ km s^{-1} and $T_B = 3.085$ s, this condition corresponds to $\cos\alpha_o \leq 6 \times 10^{-5}$ or α_o differing from 90° by only 0.004°. Within this range of α_o, $\langle p_2 \rangle$ must be set equal to 1.0 and $\langle P \rangle$ is very much greater than it is for values of α_o that differ substantially from 90°. It is therefore clear that particles having α_o near 90° are preferentially absorbed. The dependence of $\langle P(\alpha_o) \rangle$ on α_o is illustrated by Table 1 (all for $E_p = 200$ MeV).

For an angular distribution of unidirectional particle intensity of the form $\sin^n \alpha_o$, the weighted value of $<P>$ over the range $0° \leq \alpha_o \leq 85°$ (i.e., ignoring the short-lived range $85° \leq \alpha_o \leq 90°$) is equal to $<P(\alpha_o)>$ with $\alpha_o = 74°$ to $77°$ for n = 1, 2, and 3; and over the range $0° \leq \alpha_o \leq 80°$, the weighted value of $<P>$ is equal to $<P(\alpha_o)>$ with $\alpha_o = 64°$ to $70°$ for n = 1, 2, and 3. These calculations provide the justification for adopting $\alpha_o = 70°$ as a representative value for simplified considerations which do, of course, ignore refined features of the absorption process (i.e., the angular dependence of $<P(\alpha_o)>$). It is noted that the absorption signatures of Ring G were measured with omnidirectional detectors.

If the magnetic axis of the planet is tilted with respect to its rotational axis, and/or if the orbit of the hypothetical satellite is inclined to the planet's equatorial plane, there are additional considerations. However, if the tilt and inclination angles are small ($\lesssim 1°$), then the consequent effects are significant only for α_o near 90° and are otherwise swallowed up in the distribution of particle intensity with α_o. For Saturn, the tilt of the magnetic axis is $\lesssim 1°$ [Connerney et al., 1984] and the inclination of the orbits of known inner satellites is also $\lesssim 1°$ [Synnott et al., 1981, 1983]. Further, if the magnetic dipole moment of the planet is offset from the geometrical center of the planet and/or

if the hypothetical satellite is moving in an eccentric orbit, the effect is to reduce the probability of a hit per T_E for a specified particle. The ratio r_g/r ($\approx$ 0.0135) gives a measure of the magnitude of the combined eccentricities at which this effect becomes significant. By the references just cited, the offset of the dipole is < 600 km (i.e., < 0.26 r_g) and the eccentricities of the orbits of the known inner satellites are $\leq$ 0.009. Hence, the potential defects in the simplified derivation of equation (13) as listed in this paragraph are probably trivial. Observational support for this conclusion is provided by Figures 4 and 5 of Van Allen [1983], which show that the radial width of the absorption signature of Ring G is, in fact, not significantly greater than 2 r_g. The guiding centers of particles do, of course, diffuse back and forth in r; if the characteristic time constant for such diffusion is less than the lifetime for absorption, then the signature is broadened. The above observational fact apparently shows that this effect is also not important -- a result that is more persuasive than, but consistent to an order of magnitude with, an estimate of the time constant for diffusion, which is $r_g^2/4D \approx 7$ months.

In Van Allen [1982] it is shown that the energy distribution of protons in the vicinity of Ring G is such that about half of the particles have energy E_p greater than 200 MeV, and half have energy

E_p less than 200 MeV. This is the foundation for adopting 200 MeV as a representative value of E_p in the foregoing numerical examples.

Overall, I estimate that the numerical values in equations (14) and (16) are trustworthy to a factor of two.

ADDITIONAL COMMENTS

A noteworthy feature of equation (13) is that, for b sufficiently small (i.e., $\leq$ 10 km), $\langle P \rangle$ is proportional to the sum of the cross-sections of the assemblage of absorbing satellites as written in (15). The latter equation is applicable to particulate matter in the form of small spheres of radius b and volumetric mass density ρ provided that $4 \rho b/3$ is greater than the range of the relevant protons ($\approx$ 26 g cm^{-2}) in the particulate material. If a sphere is pulverized into n equal smaller spheres and if the latter are dispersed widely (no "shadowing"), then the projected area of the assemblage is increased by the factor $n^{1/3}$. If, however, the radii of the small objects are such that $4 \rho b/3$ is much less than the range of the protons, a given quantity of material becomes an even more effective absorber. The latter case is found to be applicable to the particulate matter in Ring G, if its absorption signature is attributed entirely to small particulates of uniform size [Van Allen, 1983]. The inferred sum of the volumes of all particulates in the ring is equivalent to the volume of a single sphere of radius only 0.1 km. The inferred total number of particulates is 4.3×10^{16}. By the pulverization theorem, the

radius of a single satellite having the same absorption for energetic protons would be 8 km <u>if</u>, contrary to my previous analysis, the radii b of the particulates were greater than $4\rho b/3$.

For a single satellite with $b \geqslant 100$ km, $\langle p_1 \rangle$ must be set equal to unity for each of the two opportunities during an encounter. Then

$$\langle P \rangle = 2 \langle p_2 \rangle = \frac{2\pi b}{V T_B \cos \alpha_o} \tag{19}$$

or numerically

$$\langle P \rangle = 3.5 \times 10^{-5}\ b \tag{20}$$

for $\alpha_o = 70°$ and b in km. In this case, $\langle P \rangle$ is proportional to the first power of b and, corresponding to (16) and (17)

$$\tau = \frac{196}{b} \text{ years} \tag{21}$$

for a single satellite or

$$\tau = \frac{196}{\Sigma\, b_i} \text{ years} \tag{22}$$

for more than one.

The intersection of (20) and (14) occurs at

$$b \;=\; 100 \text{ km}. \tag{23}$$

There is a gradual transition from the quadratic regime for $b \leq 20$ km to the linear regime for $b \geq 100$ km. As b increases further, $\langle P \rangle$ approaches 1.0 asymptotically.

CONCLUSIONS

The analysis in Van Allen [1983] finds that the lifetime against absorption of 200 MeV protons whose guiding centers lie within $\pm r_g$ of the center of Ring G lies in the range 1.6 to 3.5 years. If one adopts an intermediate value of 2.5 years, then by (16)

$$b \approx 3.3 \text{ km} \tag{24}$$

or by (17)

$$\sum_{i=1}^{n} b_i^2 = 11.2 \text{ km}^2 . \tag{25}$$

The reader is reminded that these estimates are in the nature of extreme upper limits for the radii of one or more shepherding satellites because they ignore the absorbing effects of the optically-observed distribution of particulate matter.

It is difficult to assign a rigorous uncertainty to the foregoing values but I estimate that equation (24) is uncertain by less than a factor of two and equation (25), by less than a factor of four.

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REFERENCES

Acuña, M. H., and N. F. Ness, The magnetic field of Saturn: Pioneer 11 observations, Science, 207, 444-446, 1980

Acuña, M. H., N. F. Ness, and J.E.P. Connerney, The magnetic field of Saturn: Further studies of Pioneer 11 observations, J. Geophys. Res., 85, 5675-5678, 1980

Bell II, E. V., and T. P. Armstrong, Monte Carlo simulation of charged particle impact on the satellites of Jupiter and Saturn, J. Geophys. Res., 91, 1397-1403, 1986

Connerney, J.E.P., L. Davis, Jr., and D. L. Chenette, Magnetic field models. In Saturn, ed. by T. Gehrels and M. S. Matthews, pp. 354-377, University of Arizona Press, Tucson, AZ, 1984

Fillius, W., W.-H. Ip, and C. E. McIlwain, Trapped radiation belts of Saturn: First look, Science, 207, 425-431, 1980

Fillius, W., and C. McIlwain, Very energetic protons in Saturn's radiation belt, J. Geophys. Res., 85, 5803-5811, 1980

Gehrels, T., et al., Imaging photopolarimeter on Pioneer Saturn, Science, 207, 434-439, 1980

Goldreich, P., and S. Tremaine, Toward a theory for the Uranian rings, Nature, 277, 97-99,1979

Goldreich, P., and S. Tremaine, The dynamics of planetary rings, Ann. Rev. Astron. Astrophys., 20, 249-283, 1982

Hood, L. L., A comparison of characteristic times for satellite absorption of energetic protons trapped in the Jovian and Saturnian magnetic field, Geophys. Res. Lett., 8, 976-979, 1981

McDonald, F. B., A. W. Schardt, and J. H. Trainor, If you've seen one magnetosphere, you haven't seen them all: Energetic particle observations in the Saturn magnetosphere, J. Geophys. Res., 85, 5813-5830, 1980

Mead, G. D., and W. N. Hess, Jupiter's radiation belts and the sweeping effects of its satellites, J. Geophys. Res., 78, 2793-2811, 1973

Mogro-Campero, A., and W. Fillius, The absorption of trapped particles by the inner satellites of Jupiter and the radial diffusion coefficient of particle transport, J. Geophys. Res., 81, 1289-1295, 1976

Paonessa, M. T., and A. F. Cheng, A theory of satellite sweeping, J. Geophys. Res., 90, 3428-3434, 1985

Rairden, R. L., Satellite sweeping of electrons and protons in Saturn's inner magnetosphere, Res. Rep. 80-29, Univ. of Iowa, Iowa City, July 1980

Schardt, A. W., and F. B. McDonald, The flux and source of energetic protons in Saturn's inner magnetosphere, J. Geophys. Res., 88, 8923-8935, 1983

Simpson, J. A., T. S. Bastian, D. L. Chenette, G. A. Lentz, R. B. McKibben, K. R. Pyle, and A. J. Tuzzolino, Saturnian trapped radiation and its absorption by satellites and rings: The first results from Pioneer 11, Science, 207, 411-415, 1980a

Simpson, J. A., T. S. Bastian, D. L. Chenette, R. B. McKibben, and K. R. Pyle, The trapped radiations at Saturn and their absorption by satellites and rings, J. Geophys. Res., 85, 5731-5762, 1980b

Singer, S. F., Radiation belts of Venus and Mars (with consideration of sweeping effect of Phobos). In Space Age Astronomy, ed. by A. J. Deutsch and W. G. Klemperer, pp. 444-461, Academic Press, New York, 1962

Smith, E. J., L. Davis, Jr., D. E. Jones, P. J. Coleman, Jr., D. S. Colburn, P. Dyal, and C. P. Sonett, Saturn's magnetic field and magnetosphere, Science, 207, 407-410, 1980

Smith, B. A., et al., Encounter with Saturn: Voyager 1 imaging science results, Science, 212, 163-191, 1981

Smith, B. A., et al., Voyager 2 in the Uranian system: Imaging science results, Science, 233, 43-64, 1986

Synnott, S. P., C. F. Peters, B. A. Smith, and L. A. Morabito, Orbits of the small satellites of Saturn, Science, 212, 191-192, 1981

Synnott, S. P., R. J. Terrile, R. A. Jacobson, and B. A. Smith, Orbits of Saturn's F ring and its shepherding satellites, Icarus, 53, 156-158, 1983

Thomsen, M. F., On determining a radial diffusion coefficient from the observed effects of Jupiter's satellites, Ph.D. dissertation, University of Iowa, 1977

Thomsen, M. F., C. K. Goertz, and J. A. Van Allen, On determining magnetospheric diffusion coefficients from the observed effects of Jupiter's satellite Io, J. Geophys. Res., 82, 5544-5550, 1977

Thomsen, M. F., and J. A. Van Allen

Motion of trapped electrons and protons in Saturn's inner magnetosphere, J. Geophys. Res., 85, 5831-5834, 1980

Trainor, J. H., F. B. McDonald, and A. W. Schardt, Observations of energetic ions and electrons in Saturn's magnetosphere, Science, 207, 421-425, 1980

Van Allen, J. A., Findings on rings and inner satellites of Saturn by Pioneer 11, Icarus, 51, 509-527, 1982

Van Allen, J. A., Absorption of energetic protons by Saturn's ring G, J. Geophys. Res., 88, 6911-6918, 1983

Van Allen, J. A., B. A. Randall, and M. F. Thomsen, Sources and sinks of energetic electrons and protons in Saturn's magnetosphere, J. Geophys. Res., 85, 5679-5694, 1980a

Van Allen, J. A., M. F. Thomsen, B. A. Randall, R. L. Rairden, and C. L. Grosskreutz, Saturn's magnetosphere, rings and inner satellites, Science, 207, 415-421, 1980b

Table 1. Pitch-Angle Dependence of the Probability of Absorption -- Relative to That at 70°.

α_o	$\frac{\langle P(\alpha_o) \rangle}{\langle P(70°) \rangle}$	α_o	$\frac{\langle P(\alpha_o) \rangle}{\langle P(70°) \rangle}$
89°	20.36	45°	0.42
85	4.07	40	0.37
80	2.03	35	0.34
75	1.34	30	0.31
70	1.00	25	0.28
65	0.79	20	0.26
60	0.65	15	0.24
55	0.55	10	0.22
50	0.48	5	0.21

FIGURE CAPTIONS

Figures 1(a) and 1(b).

The origin of the adopted Cartesian coordinate system is the point at which the guiding center of a particular gyrating particle pierces the equatorial plane; the origin moves in longitude at the same average rate as does the guiding center. The Y-axis if directed toward the planet's center and the Z-axis is parallel to its axis of rotation. In each of the two drawings, the large circle represents the equatorial projection of the segment of the particle's helical trajectory that lies near the equatorial plane. The small circles represent successive positions of the equatorial cross-section of a spherical satellite as it moves at constant speed v_x parallel to the instantaneous X-axis with its center at $y = y_o$, $z = 0$. Note that for the cases of interest in this paper $b/r_g \leqslant 0.001$. A much larger value of this parameter, namely 0.1, was chosen for these drawings in the interest of graphical clarity.

Figure 2

A composite plot of equations (1) and (2) of the text in dimensionless form. Note that this illustrative plot is for $b/r_g = 0.01$, a value of the order of ten times those of relevance to the cases of interest in this paper.

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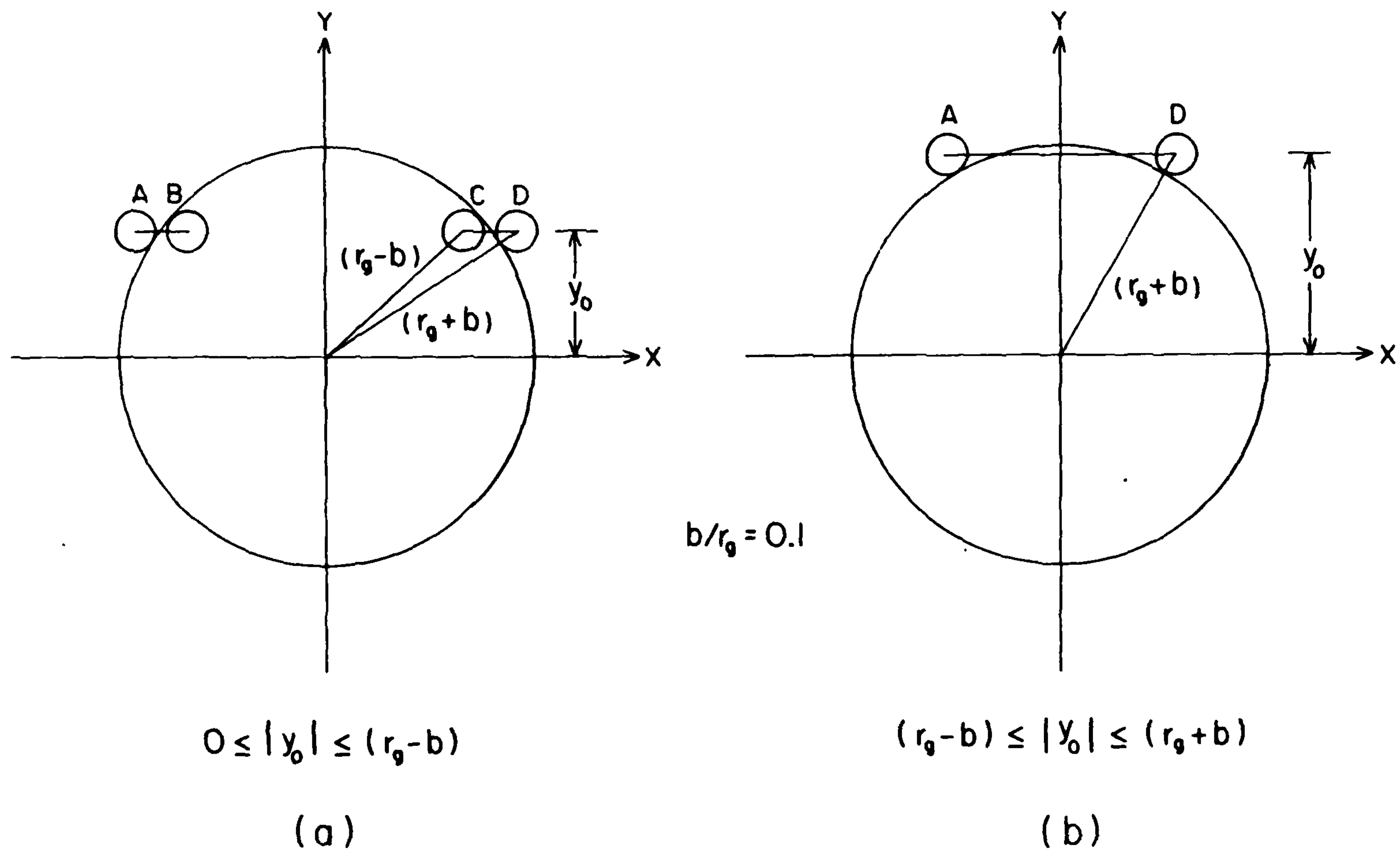

Figure 1

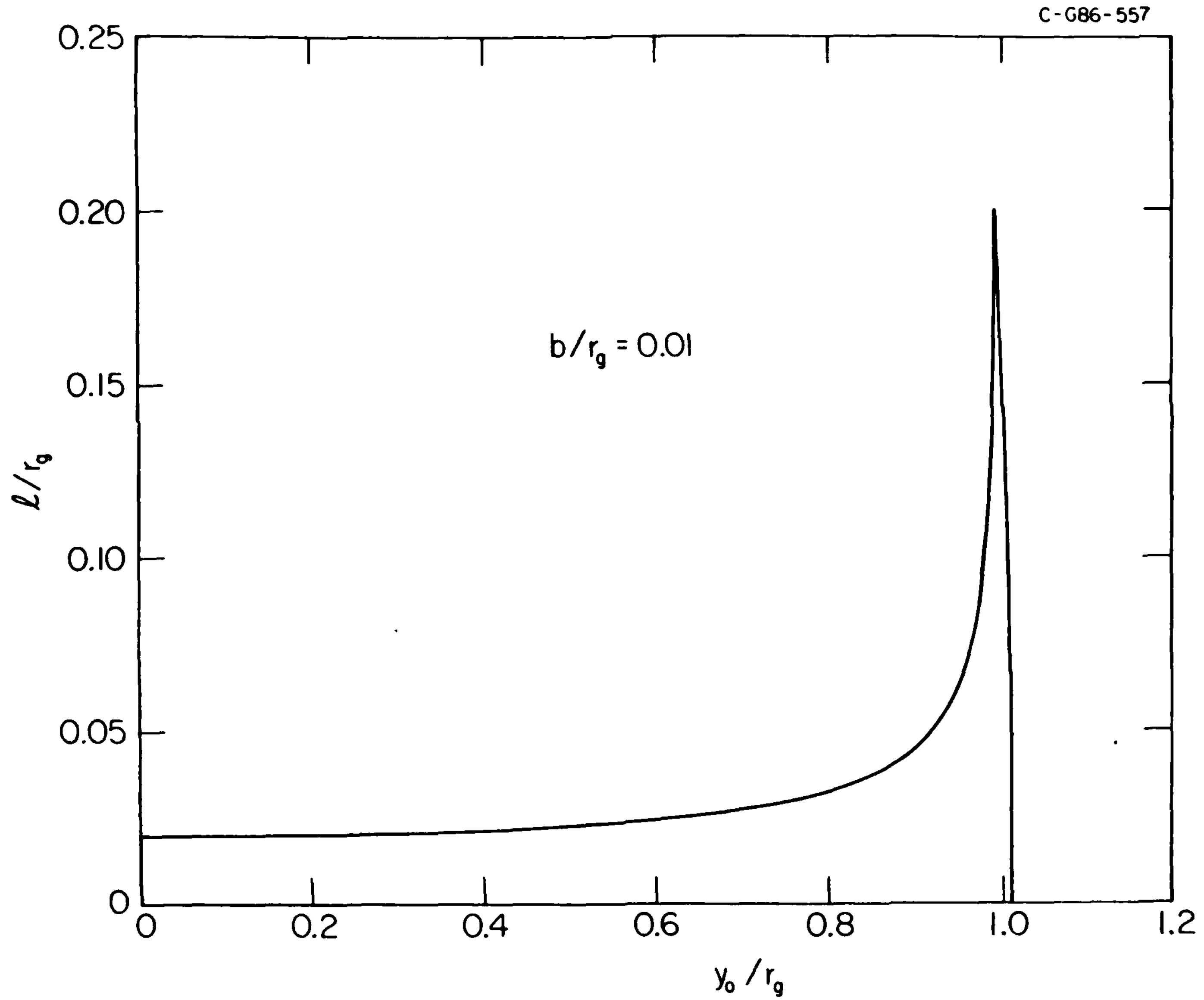

Figure 2

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